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DETERMINING THE MEAN-SQUARE ERROR AND DISCRETIZATION STEP OF THE INITIAL DATA OF AN INVERSE PROBLEM IN A SINGLE REALIZATION

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UDC 536.24.02

A method is developed allowing the approximate values of the mean square error and optimal discretization step of the initial data to be found from a single realization of a random process.

In solving inverse problems by means of information on the mean square error σ of the initial data, the accuracy of the results obtained depends on the accuracy in determining σ. The economy and accuracy of computer calculations depends largely on the number of discretization points of the initial inverse-problem data.

To determine the optimal discretization step H_{opt} of a random process $T(\tau)$ consisting of a useful signal and an arbitrarily distributed perturbation, it is assumed that the greatest frequency $T(\tau)$ is finite and $T(\tau)$ is specified by the division T_i, $i = 1, \ldots, N$, in sufficient detail (no less than three points must cover each halfperiod of the characteristic variations). Using a cubic spline $S_\Delta(\tau, T_i)$ interpolating the values of T_i, the characteristic frequency f_{max} of high-frequency oscillations of the function $T(\tau)$ with respect to the number of points N^* of sign change of the second derivative $S''_\Delta(\tau, T_i)$ on the given segment $[0, \tau_{max}]$ is found

$$f_{\max} = \frac{N^*}{2\tau_{\max}}.$$

In accordance with the Kotel'nikov and Zheleznov discretization theorem — see [1], for example — the division $S_\Delta(\tau, T_i)$ is made with a step equal to half the characteristic period of the high-frequency oscillations, that is, with

$$H_{\text{opt}} = \frac{1}{2f_{\max}}.$$

This step is very close to the maximum possible value at which all the information on the useful signal and the error of the initial data $T(\tau)$ is retained. To determine the mean square error σ of the initial data $T(\tau)$, the squares of the deviations of $S_\Delta(\tau, T_i)$ at each internal point of the chosen optimal grid division from the straight lines passing through two adjacent corners are averaged. This leads to the value

$$\delta^2 = \frac{1}{N^* - 1} \sum_{i=2}^{N^*} \left[T_i^* - \frac{1}{2} (T_{i+1}^* - T_{i-1}^*) \right]^2,$$

where T_i^*, $i = 1, \ldots, N^*$, are the corner values of the optimal grid division. To determine the difference of δ from σ, the error of the initial data is specified using a harmonic function of the form

$$\varepsilon(\tau) = a \sin(2\pi f_{\max}\tau).$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 1, pp. 150-152, July, 1986. Original article submitted May 17, 1985.

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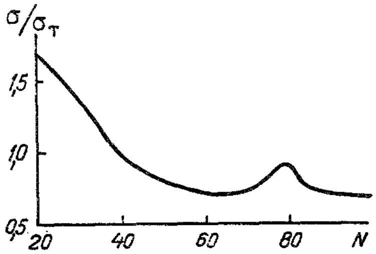

Fig. 1. Ratio of the results obtained for the mean square error of the initial data and the accurate values, as a function of the size of the statistical sample employed.

In this case, δ^2 is equal to the mathematical expectation of the function $[a+\varepsilon(\tau)]^2$, and σ^2 is the dispersion of the function $\varepsilon(\tau)$ on the segments $[0, 1/f_{max}]$. Hence $\sigma^2 = \delta^2/3$. This relation is used to estimate the mean square error of the initial data $T(\tau)$, under the assumption that the basic contribution to the error comes from high-frequency oscillations.

The results of verifying this procedure for determining σ on a model example are shown in Fig. 1. The value of σ, referred to the known accurate value of the mean square error σ_T, is shown as a function of the size N of the statistical sample describing the given random process. The initial function chosen is

$$T(\tau) = \sin\tau,\ 0 \leqslant \tau \leqslant 3\pi,$$

with the node values T_i, i = 1, ..., N, which introduce a perturbation distributed according to a normal law. The mean square error σ_T of these perturbations is 1% of the range of variation of T. As shown by numerical experiment, the size of the statistical sample has the greatest influence on the accuracy of determination of σ at small N. Therefore, to increase the accuracy of a calculation when $N \lessgtr 30$, initial grid values of T_i must be used instead of T_i^*, since $N \geqslant N^*$.

NOTATION

T, function of the initial data; S_Δ, cubic spline; H_{opt}, optimal discretization step; f_{max}, characteristic frequency of high-frequency oscillations; δ, characteristic magnitude of the error of the initial data; σ, mean square error; σ_T, accurate value of mean square error; ε, model error function; τ, independent variable; τ_{max}, maximum value of independent variable; N, number of points of initial grid division; N*, number of points of optimal grid division; i, number of point.

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